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# BOUNDARY CONDITIONS FOR THE HEAT- AND MASS-TRANSFER EQUATIONS OF COARSELY DISPERSE AEROSOLS IN A TURBULENT FLOW

I. V. Derevich and V. M. Eroshenko

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Boundary conditions taking account of particle interaction with the boundary surface are obtained on the basis of the Chapman–Enskog method of solving the kinetic equation.

In the turbulent flow of coarsely disperse aerosols in channels, the dynamic relaxation time of the particles considerably exceeds the lifetime of the energy-content pulsations of the carrying flux. In this case, processes occurring in the interaction of the discrete phase with the surface exert a significant influence on the dynamic and thermal characteristics of the disperse flow. In calculations of the turbulent disperse flows, the collision of particles with the walls is taken into account by formulating the corresponding boundary conditions for the hydrodynamic and heat- and mass-transfer equations. In [1-4], the boundary condition for the concentration of Brownian particles at a partially absorbing surface was constructed. The boundary concentration for the concentration of disperse impurity in the turbulent flow, taking account of inhomogeneity of the turbulent-pulsation field, the mass force, and the degree of particle entrainment in the turbulent motion, was found in [5]. The boundary conditions of [1-5] are conditions of the third kind for the particle-diffusion equations and relate the concentration value and its gradient at the surface. The distinguishing feature of these boundary conditions is the nonzero particle concentration at an absolutely absorbing wall.

The distribution of the pulsational characteristics of inertial particles in an inhomogeneous turbulent flow is determined by the ratio between the scale of inhomogeneity of the pulsational field of the carrier phase and the pulsational inertial path length of the particle $\ell_p = \tau_u \sigma^{1/2}$ [6, 7]. If the inertial path length of the particle is comparable with the characteristic scale of inhomogeneity of the turbulent pulsations of the fluid phase close to the channel wall, the intensity of pulsational motion of the discrete phase in the wall region is determined by the pulsational energy of the particle acquired in the flow core. In this case, the intense turbulent motion of particles around the wall leads to effective turbulent transfer of the mean flow characteristics (axial velocity, temperature) from the flow core to the surface. The description of such flow must be based on the two-velocity and two-temperature approximation, when the equations for the particle velocity and temperature are used together with the equations for the mean velocity and temperature of the carrier phase. On account of the intense transverse turbulent transfer in the solid phase between the flow core and the wall region, the equations for the mean characteristics of the discrete phase are of diffusional type. Accordingly, the formulation of boundary conditions taking account of the processes occurring at particle contact with surface is an urgent problem.

In the present work, a closed system of equations and boundary conditions for calculat-

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ing heat- and mass-transfer processes in the discrete phase is constructed on the basis of the kinetic equation for the probability density function (PDF) of the particle distribution with respect to the coordinate, velocity, and temperature.

Consider the turbulent flow of a disperse flux with a small volume concentration of particles, when interaction between the particles may be neglected. The equations for the velocity, coordinate, and temperature of the p-th particle take the form

$$\frac{dV_{pi}}{d\tau}=\frac{1}{\tau_u}\left(U_i\left(\mathbf{R}_p(\tau),\ \tau\right)-V_{pi}\right),\quad \frac{dR_{pi}}{d\tau}=V_{pi};$$
$$\frac{d\Theta_p}{d\tau}=\frac{1}{\tau_\theta}\left(T\left(\mathbf{R}_p(\tau),\ \tau\right)-\Theta_p\right). \tag{1}$$

For coarsely disperse aerosols, $\tau_u$, $\tau_\theta \gg \tau_E$ ($\tau_E$ is the macroscopic time scale of turbulence). In this case, the turbulent pulsations of the velocity $u_i(\mathbf{x}, \tau)$ and temperature $t(\mathbf{x}, \tau)$ may be approximated by a random Gaussian process which is $\delta$-correlated in time. Using the method in [8], a closed equation for the PDF of the particles $\Phi(\mathbf{x}, \mathbf{V}, \Theta, \tau)$ is obtained

$$\frac{\partial\Phi}{\partial\tau}+V_k\frac{\partial\Phi}{\partial x_k}+\frac{1}{\tau_u}\frac{\partial}{\partial V_k}\left[\left(\langle U_k\rangle-V_k\right)\Phi\right]+$$
$$+\frac{1}{\tau_\theta}\frac{\partial}{\partial\Theta}\left[\left(\langle T\rangle-\Theta\right)\Phi\right]=\frac{\tau_E}{\tau_u^2}\langle u_iu_k\rangle\frac{\partial^2\Phi}{\partial V_i\partial V_k}+$$
$$+\frac{\tau_E}{\tau_\theta^2}\langle t^2\rangle\frac{\partial^2\Phi}{\partial\Theta^2}+\frac{2\tau_E}{\tau_\theta\tau_u}\langle tu_i\rangle\frac{\partial^2\Phi}{\partial V_i\partial\Theta}. \tag{2}$$

The mean (over the ensemble of turbulent realizations) concentration, velocity, and temperature of the discrete phase are defined as follows

$$\langle N(\mathbf{x},\ \tau)\rangle=\int d\mathbf{V}\int_{-\infty}^{\infty}d\Theta\Phi,\quad \langle N\rangle\langle\mathbf{V}(\mathbf{x},\ \tau)\rangle=\int d\mathbf{V}\int_{-\infty}^{\infty}d\Theta\mathbf{V}\Phi,$$
$$\langle N\rangle\langle\Theta(\mathbf{x},\ \tau)\rangle=\int d\mathbf{V}\int_{-\infty}^{\infty}d\Theta\Theta\Phi.$$

After introducing the variables x, $v_i = V_i - \langle V_i\rangle$, $\theta = \Theta - \langle\Theta\rangle$, $\tau$, Eq. (2) takes the form

$$\frac{D\Phi}{D\tau}-\frac{D\langle V_i\rangle}{D\tau}\frac{\partial\Phi}{\partial v_i}-\frac{D\langle\Theta\rangle}{D\tau}\frac{\partial\Phi}{\partial\theta}+v_k\frac{\partial\Phi}{\partial x_k}+$$
$$+\frac{\langle U_k\rangle-\langle V_k\rangle}{\tau_u}\frac{\partial\Phi}{\partial v_k}+\frac{\langle T\rangle-\langle\Theta\rangle}{\tau_\theta}\frac{\partial\Phi}{\partial\theta}-$$
$$-v_k\frac{\partial\langle V_i\rangle}{\partial x_k}\frac{\partial\Phi}{\partial v_i}-v_k\frac{\partial\langle\Theta\rangle}{\partial x_k}\frac{\partial\Phi}{\partial\theta}-$$
$$-(1-\delta_{ik})\frac{\sigma_{ik}^0}{\tau_u}\frac{\partial^2\Phi}{\partial v_i\partial v_k}-\frac{2\sigma_{i\theta}^0}{(\tau_u\tau_\theta)^{1/2}}\frac{\partial^2\Phi}{\partial v_i\partial\theta}=L\Phi, \tag{3}$$

$$L=\frac{1}{\tau_u}\frac{\partial}{\partial v_i}v_i+\frac{\sigma_{ii}^0}{\tau_u}\frac{\partial^2}{\partial v_i\partial v_i}+\frac{1}{\tau_\theta}\frac{\partial}{\partial\theta}\theta+\frac{\sigma_\theta^0}{\tau_\theta}\frac{\partial^2}{\partial\theta^2};$$
$$\frac{D}{D\tau}=\frac{\partial}{\partial\tau}+\langle V_k\rangle\frac{\partial}{\partial x_k};\quad \sigma_{ij}^0=\frac{\tau_E}{\tau_u}\langle u_iu_j\rangle,$$
$$\sigma_\theta^0=\frac{\tau_E}{\tau_\theta}\langle t^2\rangle,\quad \sigma_{i\theta}^0=\frac{\tau_E}{(\tau_u\tau_\theta)^{1/2}}\langle tu_i\rangle.$$

After integrating Eq. (3) over the space of velocities and temperatures, and then multiplying Eq. (3) by $v_i$, $\theta$, $v_i v_j$, $\theta^2$, $\theta v_i$ and integrating over the space of velocities and temperatures, a system of equations for the mean concentration, velocity, and temperature and the second moments of the velocity and temperature pulsations of the discrete phase is written

$$\frac{\partial \langle N \rangle}{\partial \tau} + \frac{\partial}{\partial x_i} \langle V_i \rangle \langle N \rangle = 0; \quad (4)$$

$$\frac{\partial \langle V_i \rangle}{\partial \tau} + \langle V_k \rangle \frac{\partial \langle V_i \rangle}{\partial x_k} + \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle \langle v_i v_k \rangle}{\partial x_k} = \frac{\langle U_i \rangle - \langle V_i \rangle}{\tau_u}; \quad (5)$$

$$\frac{\partial \langle \Theta \rangle}{\partial \tau} + \langle V_k \rangle \frac{\partial \langle \Theta \rangle}{\partial x_k} + \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle \langle \theta v_k \rangle}{\partial x_k} = \frac{\langle T \rangle - \langle \Theta \rangle}{\tau_\theta}; \quad (6)$$

$$\frac{\partial \langle v_i v_j \rangle}{\partial \tau} + \langle V_k \rangle \frac{\partial \langle v_i v_j \rangle}{\partial x_k} + \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle \langle v_i v_j v_k \rangle}{\partial x_k} + \langle v_i v_k \rangle \frac{\partial \langle V_j \rangle}{\partial x_k} + \langle v_j v_k \rangle \frac{\partial \langle V_i \rangle}{\partial x_k} = \frac{2}{\tau_u} (\sigma_{ij}^0 - \langle v_i v_j \rangle); \quad (7)$$

$$\frac{\partial \langle \Theta^2 \rangle}{\partial \tau} + \langle V_k \rangle \frac{\partial \langle \Theta^2 \rangle}{\partial x_k} + \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle \langle v_k \theta^2 \rangle}{\partial x_k} + 2 \langle \theta v_k \rangle \frac{\partial \langle \Theta \rangle}{\partial x_k} = \frac{2}{\tau_\theta} (\sigma_\theta^0 - \langle \theta^2 \rangle); \quad (8)$$

$$\frac{\partial \langle \theta v_i \rangle}{\partial \tau} + \langle V_k \rangle \frac{\partial \langle \theta v_i \rangle}{\partial x_k} + \frac{1}{\langle N \rangle} \frac{\partial \langle N \rangle \langle \theta v_i v_k \rangle}{\partial x_k} + \langle \theta v_k \rangle \frac{\partial \langle V_i \rangle}{\partial x_k} + \langle v_i v_k \rangle \frac{\partial \langle \Theta \rangle}{\partial x_k} = \frac{2\sigma_{i\theta}^0}{(\tau_u \tau_\theta)^{1/2}} - \left( \frac{1}{\tau_u} + \frac{1}{\tau_\theta} \right) \langle \theta v_i \rangle. \quad (9)$$

The system in Eqs. (4)-(9) is not closed, since moments of higher order appear in the equations for the first and second moments of the pulsations of the discrete-phase characteristics. To close this system and specify the boundary conditions, Eq. (3) is solved by approximate methods developed in statistical physics, for example, the Chapman–Enskog method [9]. Note that the operator L on the right-hand side of Eq. (3) describes the interaction of particles with turbulent moles of the carrier flow and is analogous in meaning to the collision operator in the kinetic theory of gases. In the present case, the free path length of the particles is proportional to the distance covered by the particle between collisions with turbulent moles $\ell \sim \tau_E(\langle u^2 \rangle^{1/2} - \langle v^2 \rangle^{1/2})$. For small particles ($\tau \ll \tau_E$), $\langle v^2 \rangle \to \langle u^2 \rangle$ and $\ell \to 0$, i.e., the small particles trace the trajectory of the turbulent mole. Inertial particles ($\tau \gg \tau_E$) are entrained by the pulsational motion of the carrier phase to a lesser extent: $\langle v^2 \rangle \ll \langle u^2 \rangle$; in this case, $\ell \sim \tau_E \langle u^2 \rangle^{1/2} \sim \ell_E$ ($\ell_E$ is the spatial integral scale of turbulence). The Chapman–Enskog method is applicable to systems with small gradients of the mean flow parameters. For disperse turbulent flows with inertial particles, as indicated by experimental data [10-12], weak variation in the solid-phase parameters over the channel cross section is observed. Within the framework of the given assumptions, the solution of Eq. (3) is sought in the form

$$\Phi = \Phi_0 + \Phi_1 + \ldots,$$

where the first correction $\Phi_1$ is linear in the gradient of the mean flow parameters and satisfies the normalization conditions

$$\int dv \int_{-\infty}^{\infty} d\theta \Phi_1 = \int dv \int_{-\infty}^{\infty} d\theta v_i \Phi_1 = \int dv \int_{-\infty}^{\infty} d\theta v_i v_i \Phi_1 = \int dv \int_{-\infty}^{\infty} d\theta \theta \Phi_1 = \int dv \int_{-\infty}^{\infty} d\theta \theta^2 \Phi_1 = 0. \quad (10)$$

The zero approximation $\Phi_0$ is the kernel of the operator of particle collision with turbulent moles and takes the form

$$L\Phi_0 = 0, \quad \Phi_0 = \frac{\langle N \rangle}{A} \exp\left[ -\frac{v_i v_i}{2\sigma_{ii}} - \frac{\theta^2}{2\sigma_\theta} \right], \quad A = (2\pi\sigma_\theta)^{1/2} \prod_{i=1}^{3} (2\pi\sigma_{ii})^{1/2}, \quad \sigma_{ii} = \langle v_i v_i \rangle, \quad \sigma_\theta = \langle \theta^2 \rangle. \quad (11)$$

Closing the system in Eqs. (4)-(9) on the basis of the zero approximation in Eq. (11) and expressing the time and spatial derivatives of $\Phi_0$ by means of the closed (in the first approximation with respect to the PDF) system in Eqs. (4)-(9), the equation for the first correction is obtained

$$\Phi_0^{-1}L\Phi_1=\frac{1}{\sigma_{ii}}(v_iv_k-\delta_{ik}v_i^2)\frac{\partial\langle V_i\rangle}{\partial x_k}-(1-\delta_{ik})\frac{\sigma_{ik}}{\tau_u}\frac{v_iv_k}{\sigma_{ii}\sigma_{kk}}+$$
$$+\frac{v_k}{\sigma_{ii}}\left[\frac{v_i^2}{2\sigma_{ii}}-\left(\delta_{ik}+\frac{1}{2}\right)\right]\frac{\partial\sigma_{ii}}{\partial x_k}-\frac{v_k\theta}{\sigma_{kk}\sigma_\theta}\left[\sigma_{kk}\frac{\partial\langle\Theta\rangle}{\partial x_k}-\right. \tag{12}$$
$$\left.-\frac{2\sigma_{k\theta}}{(\tau_u\tau_\theta)^{1/2}}\right]+\frac{v_k}{2\sigma_\theta}\left(\frac{\theta^2}{\sigma_\theta^2}-1\right)\frac{\partial\sigma_\theta}{\partial x_k}.$$

Integrating Eq. (12) with the normalization conditions in Eq. (10), the first correction is found in the form

$$\Phi_1=\Phi_0\left\{\frac{\sigma_{ik}}{2}(1-\delta_{ik})\frac{v_iv_k}{\sigma_{ii}\sigma_{kk}}-\frac{\tau_u}{2\sigma_{ii}}(v_iv_k-\delta_{ik}v_i^2)\frac{\partial\langle V_i\rangle}{\partial x_k}-\right.$$
$$-\frac{\tau_u}{3}\frac{v_k}{\sigma_{ii}}\left[\frac{v_i^2}{2\sigma_{ii}}-\left(\delta_{ik}+\frac{1}{2}\right)\right]\frac{\partial\sigma_{ii}}{\partial x_k}+$$
$$+\left(\frac{1}{\tau_u}+\frac{1}{\tau_\theta}\right)^{-1}\frac{v_k\theta}{\sigma_{kk}\sigma_\theta}\left[\frac{2\sigma_{i\theta}}{(\tau_u\tau_\theta)^{1/2}}-\sigma_{kk}\frac{\partial\langle\Theta\rangle}{\partial x_k}\right]-$$
$$\left.-\left(\frac{1}{\tau_u}+\frac{2}{\tau_\theta}\right)^{-1}\frac{v_k}{\sigma_\theta}\left(\frac{\theta^2}{\sigma_\theta}-1\right)\frac{\partial\sigma_\theta}{\partial x_k}\right\}. \tag{13}$$

Using the expression for $\Phi_1$, the turbulent-stress tensor, the heat-flux vector, the pulsational energy-flux vector, and the intensity of the temperature pulsations arising as a result of particle interaction with turbulent modes of fluid phase are calculated

$$\langle N\rangle\langle v_iv_j\rangle=\int d\mathbf{v}\int_{-\infty}^{\infty}d\theta v_iv_j\Phi_1=\langle N\rangle\sigma_{ij}-$$
$$-\frac{\tau_u\langle N\rangle}{2}\left\{\sigma_{ii}\frac{\partial\langle V_j\rangle}{\partial x_i}+\sigma_{jj}\frac{\partial\langle V_i\rangle}{\partial x_j}-\frac{2}{3}\delta_{ij}\sigma_{ii}\frac{\partial\langle V_i\rangle}{\partial x_j}\right\}; \tag{14}$$

$$\langle N\rangle\langle v_i\theta\rangle=\int d\mathbf{v}\int_{-\infty}^{\infty}d\theta v_i\theta\Phi_1= \tag{15}$$
$$=\langle N\rangle\left(\frac{1}{\tau_u}+\frac{1}{\tau_\theta}\right)^{-1}\left\{\frac{2\sigma_{i\theta}}{(\tau_u\tau_\theta)^{1/2}}-\sigma_{ii}\frac{\partial\langle\Theta\rangle}{\partial x_i}\right\}; \tag{16}$$

$$\langle N\rangle\langle v_iv_iv_k\rangle=\int d\mathbf{v}\int_{-\infty}^{\infty}d\theta v_iv_iv_k\Phi_1=-\frac{\tau_u}{3}\sigma_{kk}(\delta_{ik}+2\delta_{ii})\frac{\partial\sigma_{ii}}{\partial x_k}\langle N\rangle;$$

$$\langle N\rangle\langle v_k\theta^2\rangle=\int d\mathbf{v}\int_{-\infty}^{\infty}d\theta v_k\theta^2\Phi_1=\langle N\rangle\left(\frac{1}{\tau_u}+\frac{2}{\tau_\theta}\right)^{-1}\sigma_{kk}\frac{\partial\sigma_\theta}{\partial x_k}. \tag{17}$$

It follows from Eqs. (5)-(8) and (14)-(17) that the equations for calculating the mean velocity and temperature of the discrete phase and the equations for the square of the velocity and temperature pulsations of the particles are of parabolic (diffusional) type. To obtain the boundary conditions for the corresponding balance equations of the mean and pulsational characteristics of the discrete phase, the sum of the fluxes of the chosen characteristic in the flow and close to the boundary surface is found [13].

The PDF of the particles close to the wall is now written in the boundary-layer approximation $\partial\langle V_x\rangle/\partial y \gg \partial\langle V_x\rangle/\partial x$ (the y-axis is directed along the normal to the surface; the x-axis coincides with the direction of flow)

$$\Phi(x,\ v,\ \theta,\ \tau) = \Phi_0 \Big\{ 1 + \Sigma_{xy} \frac{v_x v_y}{\sigma_{xx}\sigma_{yy}} -$$

$$- \frac{\tau_u}{3} \frac{v_y}{\sigma_{ii}} \left[ \frac{v_i^2}{2\sigma_{ii}} - \left( \delta_{iy} + \frac{1}{2} \right) \right] \frac{\partial \sigma_{ii}}{\partial y} + \Sigma_{y\theta} \frac{v_y \theta}{\sigma_{yy}\sigma_\theta} -$$

$$- \left( \frac{1}{\tau_u} + \frac{2}{\tau_\theta} \right)^{-1} \frac{v_y}{2} \left( \frac{\theta^2}{\sigma_\theta} - 1 \right) \frac{\partial \ln \sigma_\theta}{\partial y} \Big\}, \qquad (18)$$

$$\Sigma_{xy} = \overset{0}{\sigma}_{xy} - \frac{\tau_u}{2} \sigma_{yy} \frac{\partial \langle V_x \rangle}{\partial y},$$

$$\Sigma_{y\theta} = \left( \frac{1}{\tau_u} + \frac{1}{\tau_\theta} \right)^{-1} \left[ \frac{2\sigma_{y\theta}}{(\tau_u \tau_\theta)^{1/2}} - \sigma_{yy} \frac{\partial \langle \Theta \rangle}{\partial y} \right],$$

where i takes the values x, y, z. The PDF of the particles reflected from the surface is written in the form

$$\Phi_+(\mathbf{x},\ \mathbf{V}',\ \Theta',\ \tau) = \int d\mathbf{V} \int_{-\infty}^{\infty} d\Theta \varphi(\mathbf{V},\ \mathbf{V}';\ \Theta,\ \Theta') \Phi(\mathbf{x},\ \mathbf{V},\ \Theta,\ \tau), \quad V_y \leqslant 0. \qquad (19)$$

The function $\phi$ in Eq. (19) describes the particle interaction with the surface. Suppose that, as a result of particle collision with the wall, the momentum of the particles reflected in the i-th direction amounts to a fraction $\kappa_i$ ($0 \leq \kappa_i \leq 1$) of the total momentum of the incident particle. On account of the thermal contact of the incident particle with the surface at a temperature $\langle T_w \rangle$, there is conductive heat transfer between the particle and the wall; the temperature of the particle reflected from the wall changes by an amount $\alpha(\langle T_w \rangle - \Theta)$ ($\Theta$ is the temperature of the incident particle). In view of the foregoing, the kernel of the transformation in Eq. (19) takes the form

$$\varphi(\mathbf{V},\ \mathbf{V}';\ \Theta,\ \Theta') = \delta(V_x' - \varkappa_1 V_x)\,\delta(V_y' + \varkappa_2 V_y) \times$$
$$\times\, \delta(V_z' - \varkappa_3 V_z)\,\delta(\Theta' - \alpha \langle T_w \rangle - (1-\alpha)\Theta). \qquad (20)$$

In Eq. (20), it is taken into account that, in particle collision with the wall, the direction of particle motion is constant with respect to the x and z axes. The PDF of the reflected particles is obtained from Eqs. (19) and (20)

$$\Phi_+(x,\ v',\ \theta',\ \tau) = \frac{1}{\varkappa_1\varkappa_2\varkappa_3\varkappa_\theta A} \exp\left\{ - \frac{v_i' v_i'}{2\varkappa_i^2 \sigma_{ii}} - \frac{\theta'^2}{2\varkappa_\theta^2 \sigma_\theta} \right\} \times$$

$$\times \Big\{ 1 - \frac{v_x' v_y'}{\varkappa_1\varkappa_2 \sigma_{xx}\sigma_{yy}} \Sigma_{xy} + \frac{\tau_u}{3} \frac{v_y'}{\varkappa_2 \sigma_{yy}} \Big[ \left( \frac{v_x'^2}{2\varkappa_1^2 \sigma_{xx}} - \frac{1}{2} \right) \frac{\partial \sigma_{xx}}{\partial y} +$$

$$+ \left( \frac{v_y'^2}{2\varkappa_2^2 \sigma_{yy}} - \frac{3}{2} \right) \frac{\partial \sigma_{yy}}{\partial y} + \left( \frac{v_z'^2}{2\varkappa_3^2 \sigma_{zz}} - 1 \right) \frac{\partial \sigma_{zz}}{\partial y} \Big] - \qquad (21)$$

$$- \frac{v_y' \theta'}{\varkappa_2 \varkappa_\theta \sigma_{yy} \sigma_\theta} \Sigma_{y\theta} + \left( \frac{1}{\tau_u} + \frac{2}{\tau_\theta} \right)^{-1} \frac{v_y'}{2\varkappa_2} \left( \frac{\theta'^2}{\varkappa_\theta^2 \sigma_\theta} - 1 \right) \frac{\partial \ln \sigma_\theta}{\partial y} \Big\},$$

where $v_i' = V_i - \varkappa_i \langle V_i \rangle$, $i = 1, 3$; $v_2' = V_y + \varkappa_2 \langle V_y \rangle$; $\theta' = \Theta - \varkappa_\theta \langle \Theta \rangle - (1 - \varkappa_\theta) \langle T_w \rangle$, $\varkappa_\theta = 1 - \alpha$.

Using Eqs. (18) and (21), the incident and reflected particle, momentum, and heat fluxes are calculated, as well as the fluxes of the square of the discrete-phase velocity and temperature. Equating the total incident and reflected fluxes, a system of boundary conditions for the numerical concentration, mean velocity, and temperature of the particles and the square of the pulsations of the normal velocity and temperature component of the discrete phase is written

$$\left( \frac{2\sigma_{yy}}{\pi} \right)^{1/2} \frac{1 - \varkappa_2}{1 + \varkappa_2} \langle N \rangle = - \langle V_y \rangle \langle N \rangle, \qquad (22)$$

$$\langle V_y \rangle = \langle U_y \rangle - \frac{\tau_u}{\langle N \rangle} \frac{\partial \sigma_{yy} \langle N \rangle}{\partial y},$$

$$\left\{ \langle V_y \rangle + \frac{1 - \varkappa_1\varkappa_2}{1 + \varkappa_1\varkappa_2} \left( \frac{2\sigma_{yy}}{\pi} \right)^{1/2} \right\} \langle V_x \rangle = -\Sigma_{xy}, \qquad (23)$$

$$\left\{ \langle V_y \rangle + \frac{1-\varkappa_2\varkappa_\theta}{1+\varkappa_2\varkappa_\theta} \left( \frac{2\sigma_{yy}}{\pi} \right)^{1/2} \right\} ( \langle \Theta \rangle - \langle T_w \rangle ) = -\Sigma_{y\theta}, \tag{24}$$

$$\left\{ \langle V_y \rangle + 2\,\frac{1-\varkappa_2^3}{1+\varkappa_2^3} \left( \frac{2\sigma_{yy}}{\pi} \right)^{1/2} \right\} \sigma_{yy} = \tau_u \sigma_{yy} \frac{\partial \sigma_{yy}}{\partial y}, \tag{25}$$

$$\left\{ \langle V_y \rangle + \frac{1-\varkappa_2\varkappa_\theta^2}{1+\varkappa_2\varkappa_\theta^2} \left( \frac{2\sigma_{yy}}{\pi} \right)^{1/2} \right\} \sigma_\theta = \left( \frac{1}{\tau_u} + \frac{2}{\tau_\theta} \right)^{-1} \sigma_{yy} \frac{\partial \sigma_\theta}{\partial y}. \tag{26}$$

The boundary conditions of the third kind in Eqs. (22)-(26) relate the values of the desired functions and their gradients at the surface. On account of the inertia of the particles, the pulsational characteristics of the disperse phase, unlike those of the fluid phase, are not zero at the boundary surface. Note that, at an absolutely absorbing surface ($\kappa_i = 0$), the particle concentration and the mean velocity of the solid phase are nonzero. As is evident from Eq. (24), the inertial particle transfer to the wall means that the temperature of the discrete phase at the surface is not $\langle T_w \rangle$ even when the particles reflected from the surface leave in the flux at the surface temperature ($\alpha = 1$, $\kappa_0$ - 0). In the case of an absolutely reflecting surface ($\kappa_i = \kappa_\theta = 1$), there is no particle flux ($\langle V_y \rangle = 0$), momentum flux ($\Sigma_{xy} = 0$), or heat flux ($\Sigma_{y\theta} = 0$); likewise, the intensity flux of the velocity pulsations $\sigma_{yy}\partial\sigma_\theta/\partial y = 0$ and the square of temperature pulsations $\sigma_{yy}\partial\sigma_\theta/\partial y = 0$ for the solid phase at the wall.

Thus, the system in Eqs. (4)-(9), together with Eqs. (14)-(17) and the boundary conditions in Eqs. (22)-(26), represents a closed description of the hydrodynamics and heat and mass transfer in turbulent fluxes of inertial particles.

## NOTATION

$\tau_u$, $\tau_\theta$, dynamic and thermal relaxation times of particles; $\sigma_{ij}$, second moment of velocity pulsation of discrete phase; $\mathbf{V}_p(\tau)$, $\mathbf{R}_p(\tau)$, $\Theta_p(\tau)$, actual velocity, coordinate, and temperature of p-th particle; $U(x, \tau)$, $T(x, \tau)$, actual velocity and temperatures of carrier flow; $\tau_E$, time scale of turbulence; $\Phi(x, V, \Theta, \tau)$, probability density function of particle distribution with respect to the coordinate, velocity, and temperature; $\langle U \rangle$, $\mathbf{u}(x, \tau)$, $\langle T \rangle$, $t(x, \tau)$, mean and pulsational components of carrier-phase velocity and temperature; $\langle V(x, \tau) \rangle$, $v$, $\langle \Theta(x, \tau) \rangle$, $\theta$ mean and pulsational components of the discrete-phase velocity and temperature; $\kappa_i$, recovery coefficient of particle momentum in impact at wall; N, number of particles in flow volume.

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